Inverse Finite Element model for the large scale ocean circulation

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The problem

• analyze the hydrographic data taking into account the ocean dynamics

» data is on irregular locations
• a model which is well suited for irregular grids
• a method of data assimilation
• we use Finite Element ocean model (FEOM) to represent the ocean dynamics
  ➢ since the data are on irregular locations

• we use the adjoint method of assimilation
  ➢ minimize the cost function which penalizes the deviation from data and model equations
Model equations
standard notation is used

- equation of state \( \rho = \rho(T, S, P) \)
- steady state primitive equations
  - dynamical part \( (u, v, w, \zeta) = \Psi(\rho, \tau) \)
    - no advection of momentum
    - strong constraint
  - equation for density
    \[ \nabla \cdot (\tilde{u} \rho) - \nabla \cdot (K \nabla \rho) = F_\rho \]
    - weak constraint
The data

• Temperature and salinity
  ✓ observations
  ✓ atlas

• Wind stress
  ✓ observations
  ✓ atlas

• Sea Surface height
  ✓ observations
Control Parameters
varied to find the best fit between the data and model equations

• density
• wind stress
Cost function

\[
J = F_\rho^T W_\rho F_\rho \\
+ (\rho - \rho_{atlas})^T W_1 (\rho - \rho_{atlas}) + (\rho - \rho_{obs})^T W_2 (\rho - \rho_{obs}) \\
+ (\tau - \tau_{atlas})^T W_3 (\tau - \tau_{atlas}) + (\tau - \tau_{obs})^T W_4 (\tau - \tau_{obs}) \\
+ (\zeta - \zeta_{obs})^T W_\zeta (\zeta - \zeta_{obs})
\]

we minimize the cost function having the dynamical part fulfilled.
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Adjoint method

Adjoint model

FEOM

dependent model variables $u, v, w, \zeta$

cost function $J$

optimal state?

initial state $T, S, \tau$

independent model variables $\rho, \tau$

descent direction
Iteration process
logarithmic scales

cost function contributions

\( J \)
\( F_\rho^T W_\rho F_\rho \)
\( (\rho - \rho_{atlas})^T W_1 (\rho - \rho_{atlas}) \)

number of iterations
The grid

• mesh is vertically stratified
• horizontal resolution varies

✓ 86701 3D nodes
✓ 6480 surface nodes
Atlas density
(density anomaly is plotted)

Annual mean
at 150m depth

\[ \tilde{\rho} = \rho - 1027.7 \, \frac{\text{kg}}{\text{m}^3} \]
Difference in density
(optimized minus climatology)

latitude
longitude

150m depth

\( \frac{kg}{m^3} \)
summary

• assimilation of the Wind does not make a big change in the Cost Function
• optimization of the open boundaries is needed

future plans

• assimilation of the Southern Ocean data